

A tale of two diversities : Weitzman's versus Rao's criterion

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Summary

1. Introduction
2. General *in-situ* framework
3. Principal results
4. Summary and perspectives

1. Introduction

- 2. General *in-situ* framework
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Motivations
Related literature

1. Introduction : Motivations



Can we design *in-situ* conservation policies ?

1. Introduction

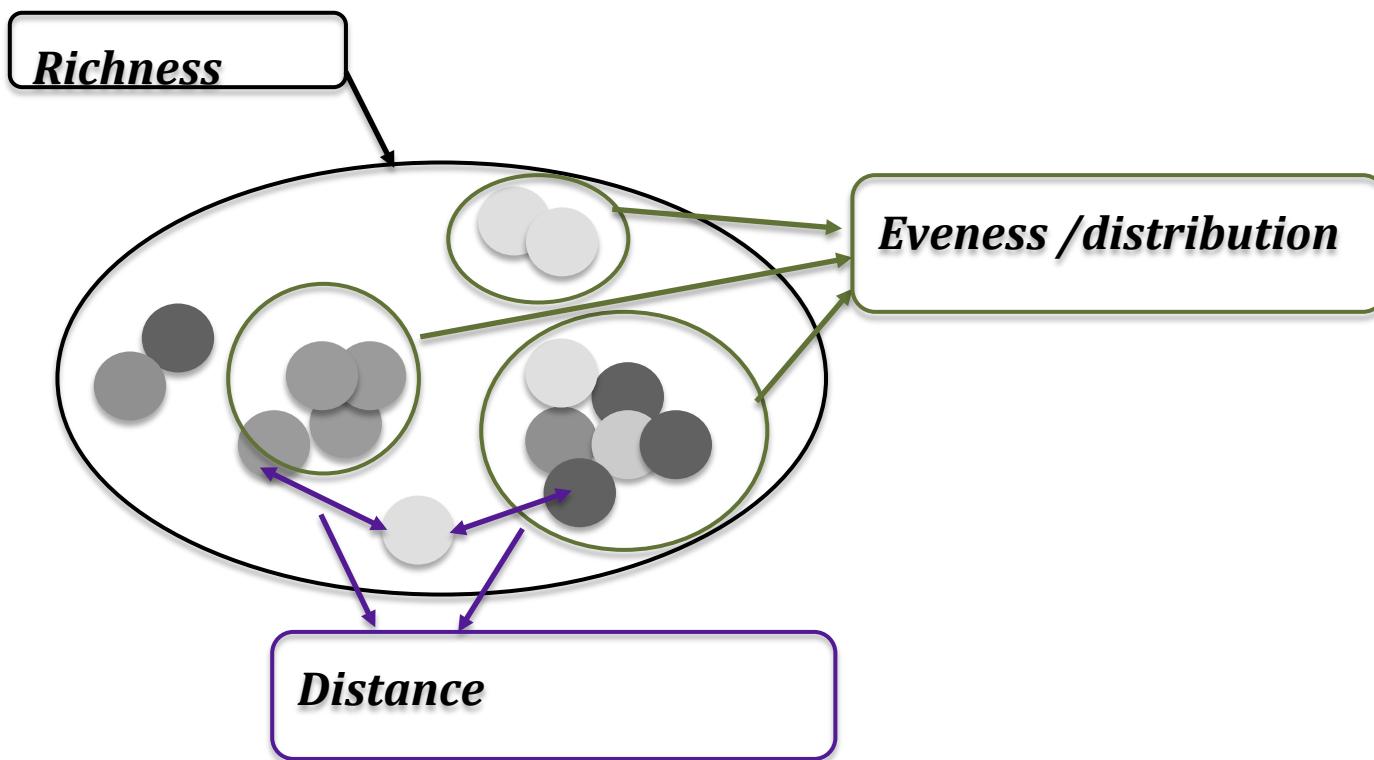
- 2. General *in-situ* framework**
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Motivations

Related literature

1. Introduction : Motivations

Ex-situ diversity indices (no interactions) combine :



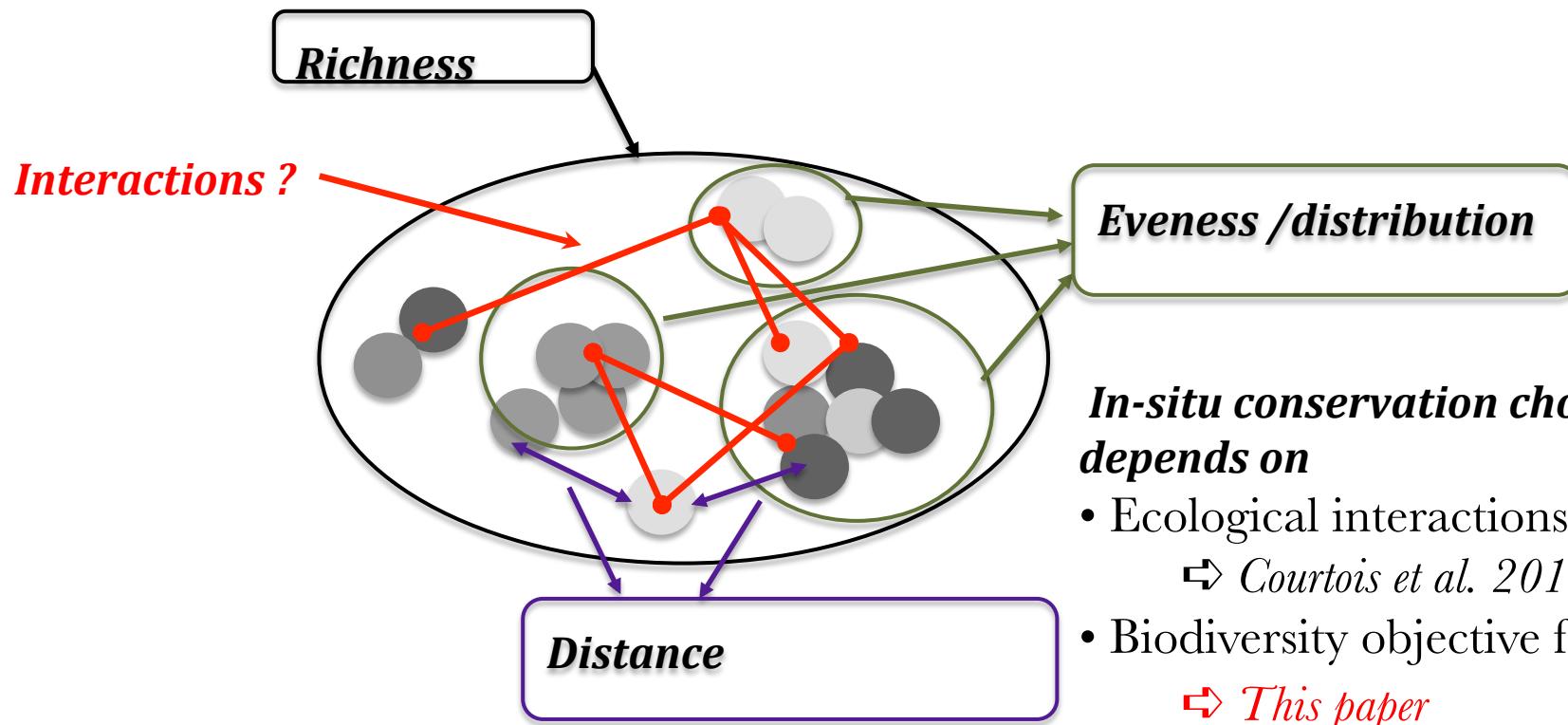
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Motivations
Related literature

1. Introduction : Motivations

With ecological interactions \Leftrightarrow *in-situ* indices



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Motivations

Related literature

1. Introduction : Related literature

1. *Biodiversity index theory*

= Measure biodiversity.

*Aulong, Figuières, Erdlenbuch
(2005, 2008)*

2. *Index Maximisation*

= Find conservation « solutions »

*Weitzman (Noah's ark problem)
(1992, 1998)*

→ **This paper builds on those 2 fronts :** ←

Compare *in-situ* conservation choices of 2 biodiversity objective functions:

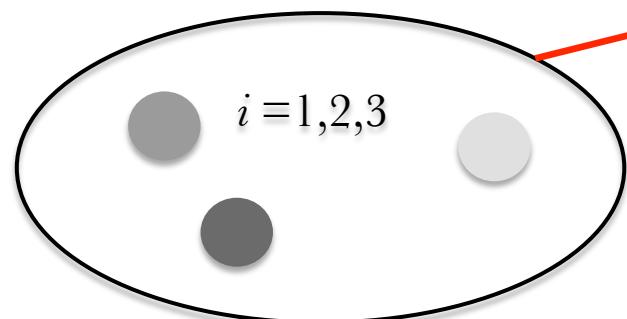
- Rao's index : $R(P)$
- Weitzman's index : $W(P)$

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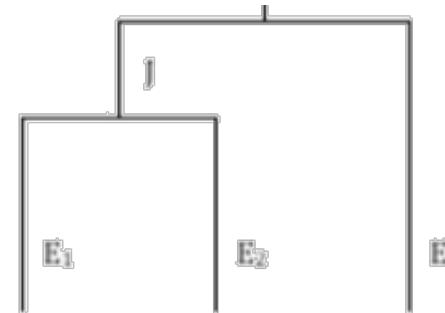
Dissimilarities
Survival probabilities
In-situ indices
In-situ protection plan

2. General *in-situ* framework : Dissimilarities

Three species ecosystem (Courtois et al 2014) :



Dissimilarities (E_i, J) :
Specific versus common « genes »

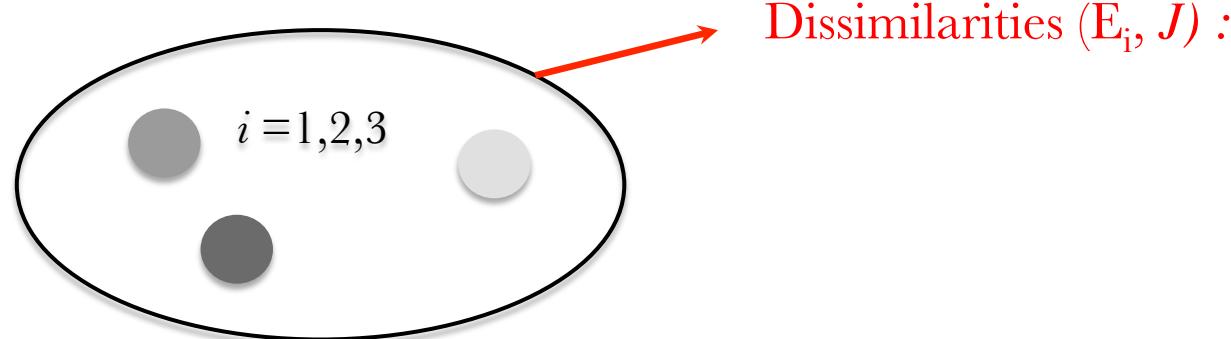


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2. General *in-situ* framework : Dissimilarities

Three species ecosystem (Courtois et al 2014) :

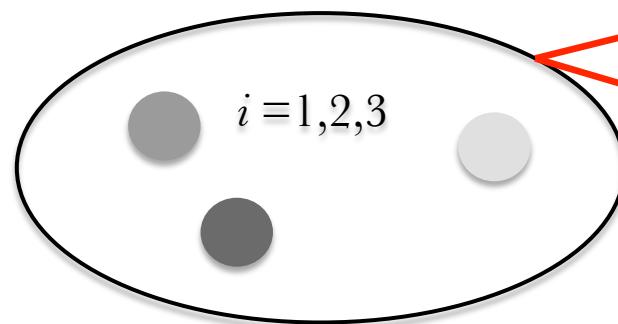


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Dissimilarities
 Survival probabilities
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2. General *in-situ* framework : Survival probabilities

Three species ecosystem (Courtois et al 2014) :



Dissimilarities (E_i, J) :

Survival probabilities P_i combining :

- q_i : autonomous survival proba
- r_{ij} : marginal ecological impact of j on i
- x_i : protection efforts on i

⇒ The system of survival probabilities :

$$P_i = q_i + x_i + r_{ih}P_h + r_{ik}P_k \implies \mathbf{Q} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}, \mathbf{R} \equiv \begin{bmatrix} 0 & r_{12} & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & r_{32} & 0 \end{bmatrix}, \mathbf{I} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \implies \mathbf{P} = \mathbf{Q} + \mathbf{X} + \mathbf{R} * \mathbf{P}$$

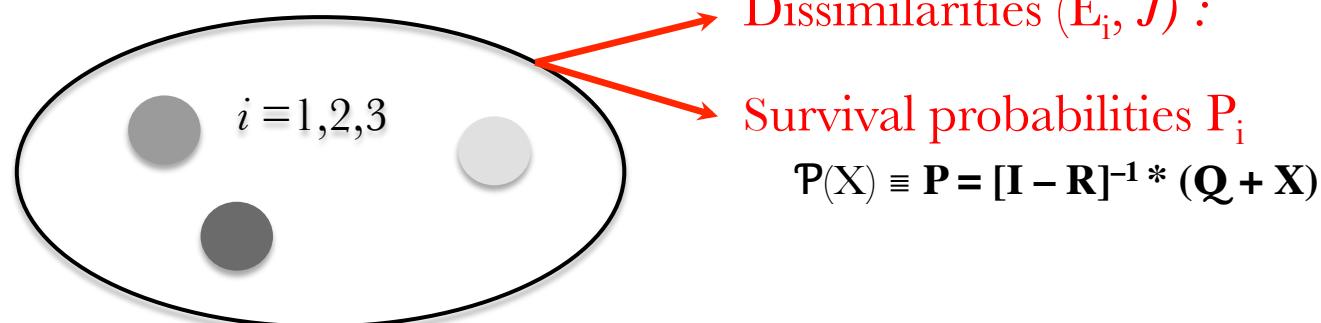
$$\mathbf{P} \equiv \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}, \bar{\mathbf{P}} \equiv \begin{bmatrix} \bar{P}_1 \\ \bar{P}_2 \\ \bar{P}_3 \end{bmatrix}, \underline{\mathbf{P}} \equiv \begin{bmatrix} \underline{P}_1 \\ \underline{P}_2 \\ \underline{P}_3 \end{bmatrix}, \mathbf{X} \equiv \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \mathbf{P}(\mathbf{X}) \equiv \mathbf{P} = [\mathbf{I} - \mathbf{R}]^{-1} * (\mathbf{Q} + \mathbf{X})$$

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2. General *in-situ* framework : Survival probabilities

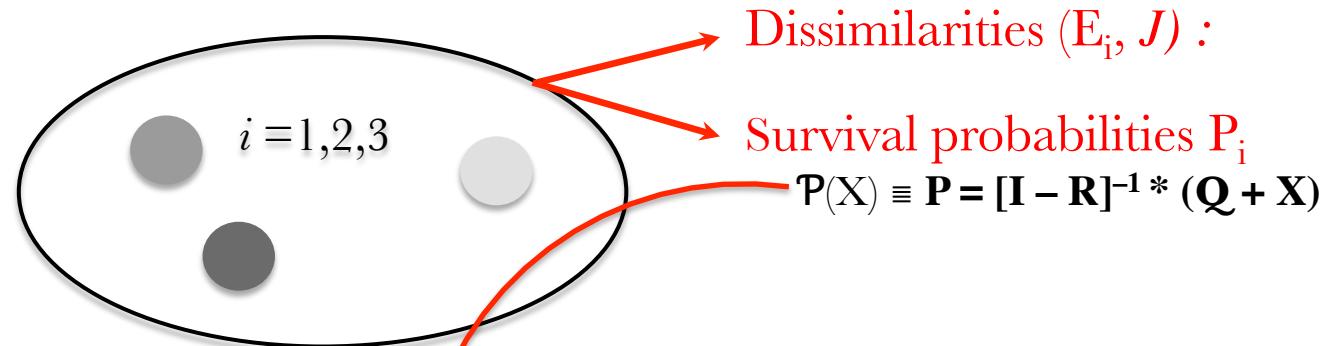
Three species ecosystem (Courtois et al 2014):



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- Survival probabilities
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2. General *in-situ* framework : *in-situ* indices



1. How to measure ? *In-situ indices for biodiversity indices*

depending on parameter $e = (\mathbf{Q}, \mathbf{R}, x, E, J) \in \Omega$

Weitzman « *ex-situ* »

$$W(P) = (E + J) * (P_1 + P_2 + P_3) - P_1 P_2 J$$

Weitzman « *in-situ* »

$$\Leftrightarrow W_e(\mathbf{X}) = \mathbf{X}^T * \mathbf{A}_e^W * \mathbf{X} + \mathbf{X}^T * \mathbf{B}_e^W + c^W$$

Rao « *ex-situ* »

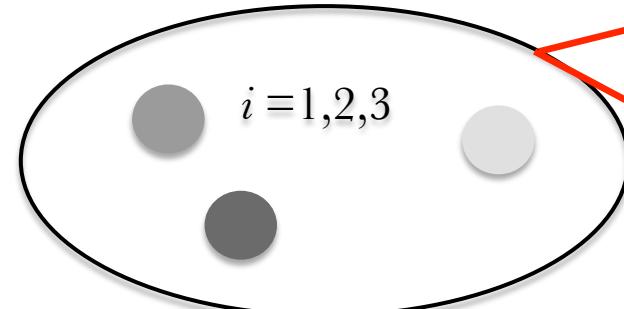
$$R(P) = 2E * [(P_1 P_2 + P_1 P_3 + P_2 P_3) + (P_1 + P_2) P_3 J] \Leftrightarrow \mathcal{R}_e(\mathbf{X}) = \mathbf{X}^T * \mathbf{A}_e^R * \mathbf{X} + \mathbf{X}^T * \mathbf{B}_e^R + c^R$$

Rao « *in-situ* »

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Survival probabilities
Dissimilarities
In-situ indices
In-situ protection plan

2. General *in-situ* framework : *in-situ* protection plans



Dissimilarity (E_i, J) :

Survival probabilities P_i

$$P(X) \equiv P = [I - R]^{-1} * (Q + X)$$

1. How to measure : In-situ indices for biodiversity

2. How to protect : Simple biodiversity protection plan

= preserving only **ONE** species out of three

$$X_1^T = [\bar{x}, 0, 0]$$

$$X_2^T = [0, \bar{x}, 0]$$

$$X_3^T = [0, 0, \bar{x}]$$

- Weitzman prefers project 1 iff :

$$\mathcal{W}_e(X_1) \geq \max \{\mathcal{W}_e(X_2), \mathcal{W}_e(X_3)\}$$

- Rao prefers project 1 iff :

$$\mathcal{R}_e(X_1) \geq \max \{\mathcal{R}_e(X_2), \mathcal{R}_e(X_3)\}$$

⇒ Study sign of

$$\mathcal{W}_{ej}(X_k) - \mathcal{W}_{ej}(X_l)$$

⇒ Study sign of

$$\mathcal{R}_{ej}(X_k) - \mathcal{R}_{ej}(X_l)$$

| | |
|--|-------------------------|
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2. General *in-situ* framework : *in-situ* protection plans

Coud we expect both indices to provide equivalent rankings of projects ?

$$\mathcal{W}_{e_j}(\mathbf{X}_k) - \mathcal{W}_{e_j}(\mathbf{X}_l) \quad \sim \quad \mathcal{R}_{e_j}(\mathbf{X}_k) - \mathcal{R}_{e_j}(\mathbf{X}_l)$$

Yes, in some cases...

No, in many other...

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When indices disagree
 When indices agree
 Interactions between effects

3. Principal results : when indices DISAGREE

Proposition 1 : Influence of autonomous survival proba (Δq_i)

Proposition 1 : 3 species, Δq_i ($q_1 \neq q_2 \neq q_3$)

► Opposite rankings :

- Weitzman indifferent :

$$\mathcal{W}_{e_q^l}(\mathbf{X}_1) - \mathcal{W}_{e_q^l}(\mathbf{X}_2) = \mathcal{W}_{e_q^l}(\mathbf{X}_1) - \mathcal{W}_{e_q^l}(\mathbf{X}_3) = 0$$

- Rao preserves the « weakest »:

$$\mathcal{R}_{e_q^l}(\mathbf{X}_1) - \mathcal{R}_{e_q^l}(\mathbf{X}_3) = \frac{2E\bar{x}}{(r+1)^2} (q_3 - q_1)$$

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When indices disagree
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3. Principal results : when indices AGREE

Proposition 2 : Influence of distinctiveness $J > 0$

Proposition 2 : 3 species, $\Delta J (J > 0)$

$$\mathbf{Q}_{e_J} = \begin{bmatrix} q \\ q \\ q \end{bmatrix}, \mathbf{R}_{e_J} = \begin{bmatrix} 0 & r & r \\ r & 0 & r \\ r & r & 0 \end{bmatrix}, \mathbf{X}_1 = \begin{bmatrix} \bar{x} \\ 0 \\ 0 \end{bmatrix}, \mathbf{X}_2 = \begin{bmatrix} 0 \\ \bar{x} \\ 0 \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} 0 \\ 0 \\ \bar{x} \end{bmatrix}, J > 0$$

► Same rankings :

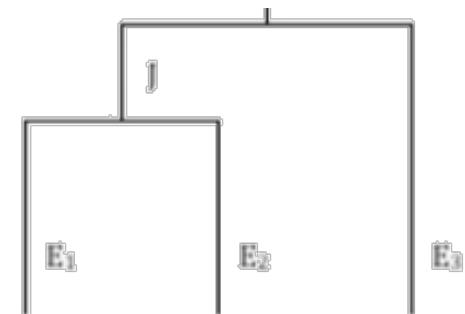
- both are indifferent between preserving the two least/equivalently dissimilar species
- both preserve the most dissimilar species (species 3)

INTUITION : With J common genes between 1 and 2

\Rightarrow the loss of 3 is greater than the loss of 2 or 1

\Rightarrow **FRAGILE** :

Results can be reversed depending on the values of r



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3. Principal results : when indices AGREE

Proposition 3 : Influence of ecological interactions (Δr_{ij})

Proposition 3 : 2 species, ΔR ($r_{12} \neq r_{21}$)

► **Same rankings** : Both preserve species with largest marginal benefit on other species

$$\begin{aligned}\mathcal{W}_{e_{R2}}(\mathbf{X}_1) &\stackrel{\cong}{\sim} \mathcal{W}_{e_{R2}}(\mathbf{X}_2) \Leftrightarrow r_{21} \stackrel{\cong}{\sim} r_{12} \\ \mathcal{R}_{e_{R2}}(\mathbf{X}_1) &\stackrel{\cong}{\sim} \mathcal{R}_{e_{R2}}(\mathbf{X}_2) \Leftrightarrow r_{21} \stackrel{\cong}{\sim} r_{12}\end{aligned}$$

Robustness : 3 species, ΔR

$$\mathbf{Q}_{e_{R3}} = \begin{bmatrix} q \\ q \\ q \end{bmatrix}, \mathbf{R}_{e_{R3}} = \begin{bmatrix} 0 & r_{12} & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & r_{32} & 0 \end{bmatrix}, \mathbf{X}_1 = \begin{bmatrix} \bar{x} \\ 0 \\ 0 \end{bmatrix}, \mathbf{X}_2 = \begin{bmatrix} 0 \\ \bar{x} \\ 0 \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} 0 \\ 0 \\ \bar{x} \end{bmatrix}$$

► **Complex conclusions :**

- Weitzman preserves 1 iff:

$$\mathcal{W}_{e_{R3}}(\mathbf{X}_1) > \max(\mathcal{W}_{e_{R3}}(\mathbf{X}_2), \mathcal{W}_{e_{R3}}(\mathbf{X}_3)) \Leftrightarrow r_{21} + r_{31} > 0$$

- Rao preserves 1 iff:

$$\mathcal{R}_{e_{R3}}(\mathbf{X}_1) > \max(\mathcal{R}_{e_{R3}}(\mathbf{X}_2), \mathcal{R}_{e_{R3}}(\mathbf{X}_3)) \Leftrightarrow r_{21}r_{31}(2q + x) + r_{31}(2q + x) + r_{21}(3q + x) > 0$$

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When indices disagree

When indices agree

Interactions between effects

3. Principal results : Interactions between effects

- Autonomous survival probability, $\Delta q_i (q_1 \neq q_2 \neq q_3)$
- Dissimilarity ($J > 0$)
 - Weitzman's prefers most distinct species β **iff $r < 1/2$**

$$W_{e'_{qJ}}(\mathbf{x}_1) - W_{e'_{qJ}}(\mathbf{x}_3) = \frac{J\bar{x} [r(q+x) + q]}{(1+r)^2(2r-1)}$$

$$W_{e'_{qJ}}(\mathbf{x}_2) - W_{e'_{qJ}}(\mathbf{x}_3) = \frac{J\bar{x} [r(q+x) + q]}{(1+r)^2(2r-1)}$$

- Rao's changes its ranking when r crosses the value $1/2$

$$R_{e'_{qJ}}(\mathbf{x}_1) - R_{e'_{qJ}}(\mathbf{x}_3) = \frac{2J\bar{x} [r(q+x) + q]}{(1+r)^2(2r-1)}$$

$$R_{e'_{qJ}}(\mathbf{x}_2) - R_{e'_{qJ}}(\mathbf{x}_3) = \frac{2J\bar{x} [r(q+x) + q]}{(1+r)^2(2r-1)}$$

⇒ **Most dissimilar species preserved if interactions r not too strong. ($r=1/2$ impossible)**

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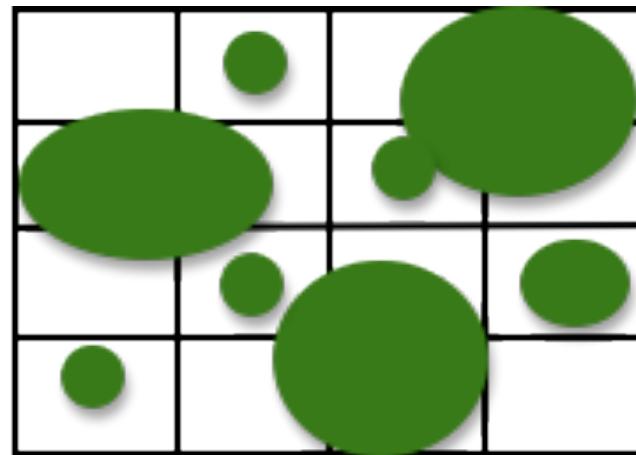
4. Summary and perspectives

- 2 *in-situ* biodiversity indices combining differently :
 - Autonomous survival probability
 - Ecological interaction coefficients
 - Data on dissimilarities
- Indices **disagree** when difference between species comes from autonomous survival probability
- Indices **agree** when differences between species comes only from dissimilarities / and or interactions.

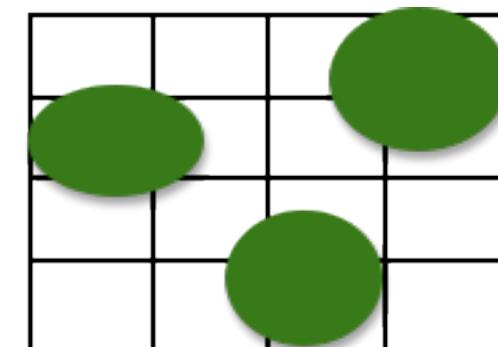
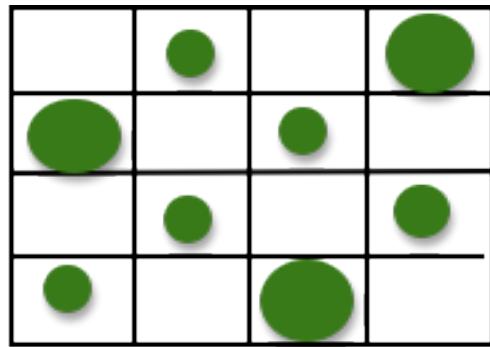
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Rao



Weitzman



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4. Summary and perspectives

Future research :

- Indirect interactions ?
- Preservation costs ?
- More complex budget constraint (preserve several instead of one species) ?
- Case study ?

